ERRATA
ARTIKEL QUANTUM INDETERMINACY
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Body, page 113-116:

The view that our world in its fundamental atomic and sub-atomic levels is highly probabilistic is caused by, for the most part, several thought-provoking counter-intuitive discoveries of quantum theory: the quantum indeterminacy and the wave property of all particles. We shall briefly discuss each of these below.

Quantum Indeterminacy

The new quantum physics gives us hints that events in the physical world are highly indeterminate. An example of this quantum indeterminacy can be given.

Many atomic nuclei and subatomic particles are unstable and, given enough time, they will decay. Uranium-235, $({}^{235}\mathrm{U})$, one of the three isotopes of Uranium, for example, is an unstable nuclei and will decay into lighter nuclei, Thorium-231 (${}^{231}\mathrm{Th}$), which itself is an unstable nuclei, causing further chain of decays. Muon(μ), a subatomic particle, will eventually decay into a group of three particle, electron (e), neutrino (v), and anti-neutrino $(\bar{v})$. The laws of physics, however, prevent anyone to predict whether and when any particular particle will decay. We can only assign probability, stated in half-life, for the decay to happen within a certain period of time. Both ${}^{235}\mathrm{U}$ and μ have a half-life time of $\sim 2.2 \times 10^{16}\mathrm{s}$ (roughly 703.8 million years) and 2.2 millionth of a second $(\sim 2.2 \times 10^{-6}\mathrm{s})$, respectively. What it means is that within this half-life time period, these particles, $^{235}\mathrm{U}$ and μ alike, will have 50% chance of decaying. Suppose we start with a sample containing a large number of μ particles. While any single one of them may not decay in its half-life period, *about* (not necessarily exact) half of the original μ sample would remain with the rest having decayed within 2.2 millionth of a second. After 4.4 millionth of a second, only *about* one-quarter of the original μ sample would remain, and so on. Physicists understand that which particle decay and when is purely a matter of chance. Quantum indeterminacy results in highly probabilistic and unpredictable events, particularly in the atomic and sub-atomic levels.

In fact, in this world of atomic and sub-atomic particles, simultaneous measurements of certain physical properties, such as a particle's velocity and position, will always be uncertain. This fundamental uncertainty principle is termed "Heisenberg's uncertainty principle," named after Werner Heisenberg who discovered the principle in 1925. Described mathematically, the Heisenberg uncertainty principle states that $\Delta p \cdot \Delta x \geq \hbar/2$, where $\hbar \approx 1.055 \times 10^{-34}$, a very small number, is the Planck constant, the notation Δ denotes uncertainty, and p and x are the particle's momentum (a product of its mass and its velocity) and its location, respectively.[399] This seemingly harmless equation yields two

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[399] In reality, the Heisenberg's uncertainty principle applies to all objects, macroscopic as well as microscopic. However, since the Planck constant is a very small number, the effect is mostly seen in the atomic and sub-atomic levels.

implications. First, our measurement cannot be arbitrarily precise; there is a limit by which our measurement cannot be made any more precise. Second, measurement that makes one property more certain will make the other property more uncertain. In Heisenberg's own words, "the more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa."

Several important points need to be noted here. First, these uncertainties in the measurements are not the result of our measurement limitation (e.g., the width of the lines in our ruler set the uncertainty for our length-measurement; the thinner the width, the more exact and certain our measurements are, et cetera). Furthermore, these uncertainties are not a result of measurement errors (e.g., systematic and random error). Rather, these uncertainties are inherent to the fabric of our universe. Our inability to simultaneously determine, with absolute certainty and precision, simple physical properties such as velocity and position is because the particles themselves cannot have definite position and velocity at the same time. Thus, for example, no particle can end up sitting at rest for this will imply that the particle has no uncertainty in its position $(\Delta x = 0)$ and in its velocity and, hence, its momentum $(\Delta p = 0)$.

Wave Property of Particles[400]

Quantum theory also informs us that all particles exhibit both wave and particle properties. Neither one property alone can fully describe the particle's behavior. Thus, in quantum mechanics it is simply meaningless to talk about a particle's exact location. The wave property implies a particle, like wave, fills all space. Observation and measurements made on a particle will collapse its wave (wave function) and only at that time particle's localization occur and we can locate the particle. However, no definite knowledge can be obtained with regards to particle's location prior to the observation and measurement being done. Under the Coulomb interaction, an electron orbiting the nucleus of a Hydrogen (H^+, or simply, a proton) will have a ground-state (i.e., lowest state: $n = 1, l = 0, m = 0$) wave function described, in spherical coordinate, by the following equation:

$$\psi_{n=1,l=0,m=0} = \psi_{100} = \frac{1}{\sqrt{\pi}}(a_0)^{-3/2}e^{-r/a_0}, \qquad (1)$$

where$a_0 \approx 5.3 \times 10^{-11}$ meter is the classical Bohr radius and r,the

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[400] The Copenhagen interpretation of quantum mechanics, pioneered by Niels Bohr and Werner Heisenberg and supported by Max Born, Wolfgang Pauli, and John von Neumann, is assumed here. It is customary in the Copenhagen interpretation to affirm only the epistemic status of the wave function, denying (or at least being agnostic) to its ontic status: the wave function is simply a mathematical tool representing information about some aspects of reality without having any discreet physical reality. However, recently it has been argued that the wave function must be real and physical (Matthew F. Pusey, Jonathan Barrett and Terry Rudolph, "On the Reality of the Quantum State," _NaturePhysics_ no. 8 (June 2012): 475-479).

radial axis on the spherical coordinate, indicates the position in space away from the nucleus (assumed to be pointlike object located at $r = 0$). It is not important for our purpose here to know how the electron wave function above is obtained. Neither it is important to understand everything in the equation. What's important is to realize that since the wave function is a function of the radial axis, r, equation (1) above tells us that the electron's wave function fills all space. Obviously at a very far distance away from the nucleus ($r$approches infinity), the wave function diminishes $(\psi_{100}(r \to \infty) {\sim} 0)$.

Furthermore, contrary to the classical expectation in which the electron is pictured to orbit the H^+ nucleus at a definite and fix distance away from it (exactly at Bohr's radius for ground-state Hydrogen electron), in the quantum picture the electron is a cloud surrounding, even penetrate, the H^+ nucleus and extending to all space. The probability density (dP) that this electron wave function is found to occupy an infinitesimal region of space dV is given by multiplying the square of the wave function with $dV = 4\pi r^2 dr$, the infinitesimal unit volume (in spherical coordinate):

$$dP = |\psi_{100}|^2 dV = \left(\frac{1}{\sqrt{\pi}}(a_0)^{-3/2} e^{-r/a_0}\right)^2 (4\pi r^2 dr)$$

$$dP = 4r^2 (a_0)^{-3} e^{-2r/a_0} dr. \qquad (2)$$

Obviously, when we integrate this probability density for all possible value of r we obtain:

$P = \int_0^\infty dP = \int_0^\infty 4r^2 (a_0)^{-3} e^{-2r/a_0} dr = 1$, which is the same as saying we will always find the electron somewhere in space. Nothing out of extra-ordinary here. But the probability of finding an electron within a region of space delimited by r_1 and $r_2 (r_1 \leq r \leq r_2)$ is given by $P(r_1, r_2) = \int_{r_1}^{r_2} 4r^2 (a_0)^{-3} e^{-2r/a_0} dr$, with a value of less than one ($P(r_1, r_2) < 1$). In other words, since we can only talk about probability of discovering the electron located in any definite region of space and that probability is less than one, we can never be certain, even after repeated measurements, that we will find the electron located in any specific region of space. The *most probable* location for the electron is given by taking the derivative of equation (2) and set it to zero (finding the maxima of the wave function), the solution of which is a_0, the classical Bohr radius, as expected. But *there is no necessity* that the ground-state electron of Hydrogen atom will be at the Bohr radius, as predicted by classical physics.

<u>Footnote 19, page 117</u>:

[19] The wave function above was obtained when only the Coulomb interaction (between the electron and the proton nucleus) is considered. This model neglects any other effects (such as relativistic effect, spin effects, and so on) and any overlap between the electron's and the proton's wave functions; In other words, to get the wave function description of the electron, the simple model starts with the assumption that the electron and proton are particles with definite localization without any wave property! But even after all the relativistic and spin-orbit coupling corrections have been applied, the

wave function is still an approximation. While it is true that the Coulomb interaction is the strongest interaction between the electron and the proton in a Hydrogen atom ($\sim 1.5 \times 10^{39}$ or 1,500 billion billion billion billion times stronger than the gravitational interaction), it is still an approximation. This model also considers the electron and proton in isolation from other particles, neglecting possible interactions with other particles. Interactions with other particles are quite small simply because other particles, even in a solid, will be an astronomical distance away compare to the distance between the electron to the nucleus. But if a particle's wave function extends all space, the wave function, as negligible as it may be, will overlap with other particle's wave function.

Body, last paragraph on conclusion, page 124::

Quantum indeterminacy may be indeterminate for us, but not so for God. In fact, God not only knows which particular ^{235}U atom will decay, but He ordains which of them should decay. He not only knows where, upon human measurement, the electron will appear to us, but He ordains where the particle will be located. Such an intervening God is the greater God!